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SOLIDIFICATION FRONTS/VISCOUS PHASE TRANSITIONS  
FORWARDS-BACKWARDS HEAT E. (U) RENSSLAER POLYTECHNIC  
INST TROY NY DEPT OF MATHEMATICAL SCIE.  
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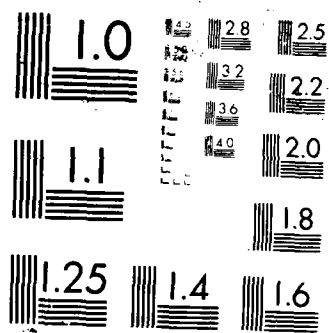
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Solidification Fronts/Viscous Phase Transitions Forwards-Backwards Heat Equations

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Directional solidification in the presence of an impurity may be described by a set of impurity concentration and thermal diffusion equations coupled at a free boundary. In the limit of a small distribution coefficient, a long wavelength expansion can be used to obtain a single fourth order parabolic equation describing the deviations of the interface from planarity in the limit in which the deviations are small. Here we present an alternate version of this asymptotic scheme which isolates and preserves the nonlinearities in their original form. While the new asymptotic expansion is of an equivalent level of asymptotic validity, the extrapolated predictions of cusping, blow up and front formation appear to be more accurate.

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**Abstract:**

1) **Solidification Fronts:** Asymptotic equations are derived to model the evolution of the solid-liquid interface which occurs in the directional solidification of binary alloys. In the limit where  $GD/VC_0(g + f_c)$  is small, the Kuramoto-Sivashinsky equation is obtained. Here  $G$  is the imposed temperature gradient,  $D$  is the diffusion coefficient of the impurity,  $V$  is the imposed transport velocity,  $g$  is equal to minus the slope of the liquidus line,  $C_0$  is the impurity concentration at the liquid side of a planar interface, and  $f_c$  is a coefficient reflecting deviations from local thermal equilibrium. The cusping and intermittancy contained in the Kuramoto-Sivashinsky equation may help to explain the creation of defect points and the onset of loss of coherency. In the limit where the segregation coefficient is small, an alternative to the Sivashinsky equation is obtained. In the new equation a quadratic nonlinearity is replaced by an exponential term. The new equation is believed to more accurately describe the solidification profile.

2) **Viscous Phase Transition:** Two equations are derived and analyzed which model the dynamics of viscous first order phase transitions. Inclusion of both viscous and gradient energy terms yields the viscous Cahn-Hilliard equation,

$$c_t = \Delta(f(c) + \nu c_t - K \Delta c) . \quad (1)$$

Here  $c(x,t)$  is a concentration,  $f(c)$  is an intrinsic chemical potential which should be nonmonotone in systems exhibiting phase separation and  $\nu$  and  $K$  are respectively coefficients of viscosity and gradient energy. In the limit  $K \rightarrow 0$ , equation (1) reduces to a viscous diffusion equation

$$c_t = \Delta(f(c)) + \nu c_t \quad (2)$$

and in the limit  $\nu \rightarrow 0$ , (1) reduces to the well-known Cahn-Hilliard equation.

$$c_t = \nu(f(c) - K\Delta c)$$

Equations (1) - (3) can be viewed as viscous and/or gradient energy regularizations of the backwards-forwards heat equation. A comparison theorem shows that early stages in the evolution of (1) follow closely the behavior of (2) for small  $K$ . The asymptotic behavior of (1) and (2) are contrasted. Equation (2) admits the possibility of discontinuous equilibria. Equation (1) initially develops transients which probably evolve and decay in much the same way as is believed to occur in the Cahn-Hilliard equation.

(ii) Report:

Solidification Fronts:

If deviations from local thermal equilibrium are taken into account, then it is possible to obtain a Kuramoto-Sivashinsky equation as a model equation for the solid-liquid interface present in the directional solidification of a dilute binary alloy. The derivation is given in a revised version of a paper which has been accepted for publication in Physica D (Appendix I). The AFOSR acknowledgement will be added in proof. In the small distribution coefficient limit, a similar derivation gives rise to a Sivashinsky equation. (Here local thermal equilibrium is assumed.) In a forthcoming paper, an alternative derivation gives rise to a similar equation in which the quadratic nonlinearity is replaced by an exponential term. This new equation is now being studied. A rough abstract of this forthcoming paper is included here (Appendix II).

Viscous Phase Transitions:

Appendix III contains a contribution to the Heriot-Watt Symposium on Non-classical Continuum Mechanics. A more detailed account of our results will appear in

- [1] Novick-Cohen, A. and Pego, R.L., Viscosity and the dynamics of phase decomposition, in preparation.
- [2] Novick-Cohen, A. and Pego, R.L., On the viscous Cahn-Hilliard equation, in preparation.
- [3] Novick-Cohen, A. and Pego, R.L., Analysis of a viscous diffusion equation, in preparation.



Asymptotic Equations in Directional Solidification: Exponential Nonlinearities

A. Novick-Cohen and P. Rosenau

Abstract: Directional solidification in the presence of an impurity may be described by a set of impurity concentration and thermal diffusion equations coupled at a free boundary. In the limit of a small distribution coefficient, a long wavelength expansion can be used to obtain a single fourth order parabolic equation describing the deviations of the interface from planarity in the limit in which the deviations are small. Here we present an alternate version of this asymptotic scheme which isolates and preserves the nonlinearities in their original form. While the new asymptotic expansion is of an equivalent level of asymptotic validity, the extrapolated predictions of cusping, blow up and front formation appear to be more accurate.

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